

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS  
SPECIALIST  
UNIT 1

**SOLUTIONS**

Section Two:  
Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 50 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 92 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (92 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (5 marks)

Determine , the vector projection of

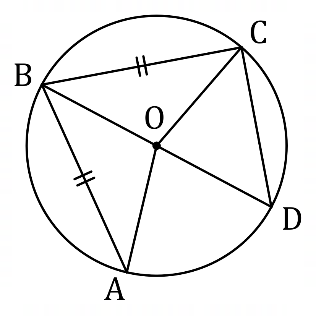
(a) a force of N on a bearing onto a force of N on a bearing of . (3 marks)

|  |
| --- |
| **Solution** |
| Hence is a force of N on a bearing of . |
| **Specific behaviours** |
| ✓ calculates angle between vectors  ü calculates magnitude  ü states direction and magnitude |

(b) on where and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates method (possibly CAS)  ü calculates vector |

Question 10 (5 marks)

Points and lie on the circle with  
centre as shown in the diagram,  
where , and  
 is a diameter.

(a) Determine the size of . (2 marks)

|  |
| --- |
| **Solution** |
| Isosceles triangle:  Angle on same arc: |
| **Specific behaviours** |
| ✓ indicates correct reasoning  ü calculates angle |

(b) Prove that . (3 marks)

|  |
| --- |
| Solution |
| Angle in semicircle:  Hence and so (corresponding sides)  Hence  *(No need to show congruency of radii, diameter, etc)* |
| **Specific behaviours** |
| ✓ establishes a pair of congruent triangles  ü establishes congruent sides or angles  ü states appropriate reason for congruency |

Question 11 (6 marks)

<EFOFEX>
id:fxd{a79b88b7-543d-41b5-8894-e97e64b541a0}

FXData:
</EFOFEX>Parallelogram is shown where point   
lies on side such that .  
  
Point , not shown, lies on diagonal   
such that .

Let and .

Express the following in terms of and .

(a) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct expression |

(b) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses ratio correctly  ü correct expression |

(c) . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses as sum of vectors  ü expresses individual vectors correctly  ü simplifies, using correct vector notation throughout |

Question 12 (8 marks)

(a) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.

(i) A quadrilateral with four congruent sides is a square. (2 marks)

|  |
| --- |
| **Solution** |
| False. Counterexample: rhombus. |
| **Specific behaviours** |
| ✓ states false  ü draws or names counterexample |

(ii) The size of one interior angle of a regular polygon with at least five sides is always obtuse. (2 marks)

|  |
| --- |
| **Solution** |
| True. Interior angle of a regular hexagon is , an obtuse angle. |
| **Specific behaviours** |
| ✓ states true  ü example with obtuse angle calculated |

(b) Consider the statement that refers to angles in triangle .

(i) Write the converse of the statement in simplest form. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct converse |

(ii) Write the contrapositive of the statement in simplest form. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ contrapositive that doesn't use 'not' |

(iii) Briefly discuss the truth of the original statement, the converse statement, and the contrapositive statement. (2 marks)

|  |
| --- |
| **Solution** |
| The original statement is true and so is the contrapositive, by definition. However, the converse is false - when the triangle is acute, for example. |
| **Specific behaviours** |
| ✓ states original and contrapositive true  ü states converse false, with justification |

Question 13 (7 marks)

(a) Points and lie on a circle of radius cm, so that is a diameter and cm. Determine the exact area of triangle . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates is right triangle  ü calculates missing side  ü calculates area |

(b) A secant meets a circle at points and , where cm. A tangent to the same circle at point intersects the secant at point , where cm. Given that , determine the exact distance and the exact distance . (4 marks)

|  |
| --- |
| **Solution** |
| Using secant-tangent theorem: |
| **Specific behaviours** |
| ✓ sketch diagram  ü formulates equation  ü solves equation for positive distance  ü calculates second distance |

Question 14 (8 marks)

A small body is acted on by force of N on a bearing of and by force of N on a bearing of .

(a) Sketch a diagram to show and their resultant . (2 marks)

|  |
| --- |
| **Solution** |
| <EFOFEX> id:fxd{cb2e8dc7-44a0-467c-a2c4-1e1379afd930}  FXData: </EFOFEX> |
| **Specific behaviours** |
| ✓ nose-to-tails vectors  ü labels and angle |

(b) Determine the magnitude and bearing of . (4 marks)

|  |
| --- |
| **Solution** |
| Magnitude of is N and bearing is . |
| **Specific behaviours** |
| ✓ expression using cosine rule with magnitude  ü calculates magnitude  ü expression using sine rule with angle  ü calculates bearing |

(c) Express in component form . (2 marks)

|  |
| --- |
| **Solution** |
| Angle of from -axis is . |
| **Specific behaviours** |
| ✓ indicates method (possibly CAS)  ü calculates components |

Question 15 (8 marks)

Consider the set of integers between and inclusive.

(a) Show that there are integers in this set that are a multiple of . (2 marks)

|  |
| --- |
| **Solution** |
| Number of multiples from to upper bound: Number of multiples from to lower bound:  Hence multiples in interval. |
| **Specific behaviours** |
| ✓ calculates multiples from to lower, upper bounds  ü calculates difference |

(b) Determine the number of integers in this set that are

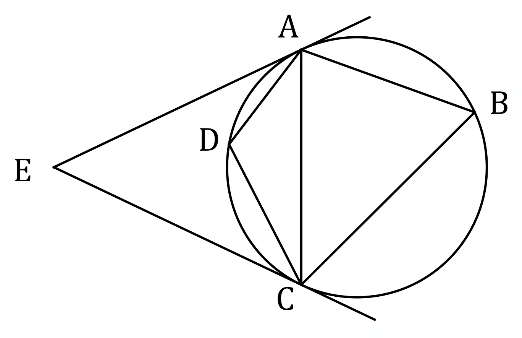
(i) a multiple of and a multiple of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states LCM  ✓ calculates multiples from to lower, upper bounds  ü calculates difference |

(ii) not a multiple of and not a multiple of . (3 marks)

|  |
| --- |
| **Solution** |
| Multiples of :  Multiples of or :  Number of integers: |
| **Specific behaviours** |
| ✓ multiples of  ü multiples of or  ü correct number of integers |

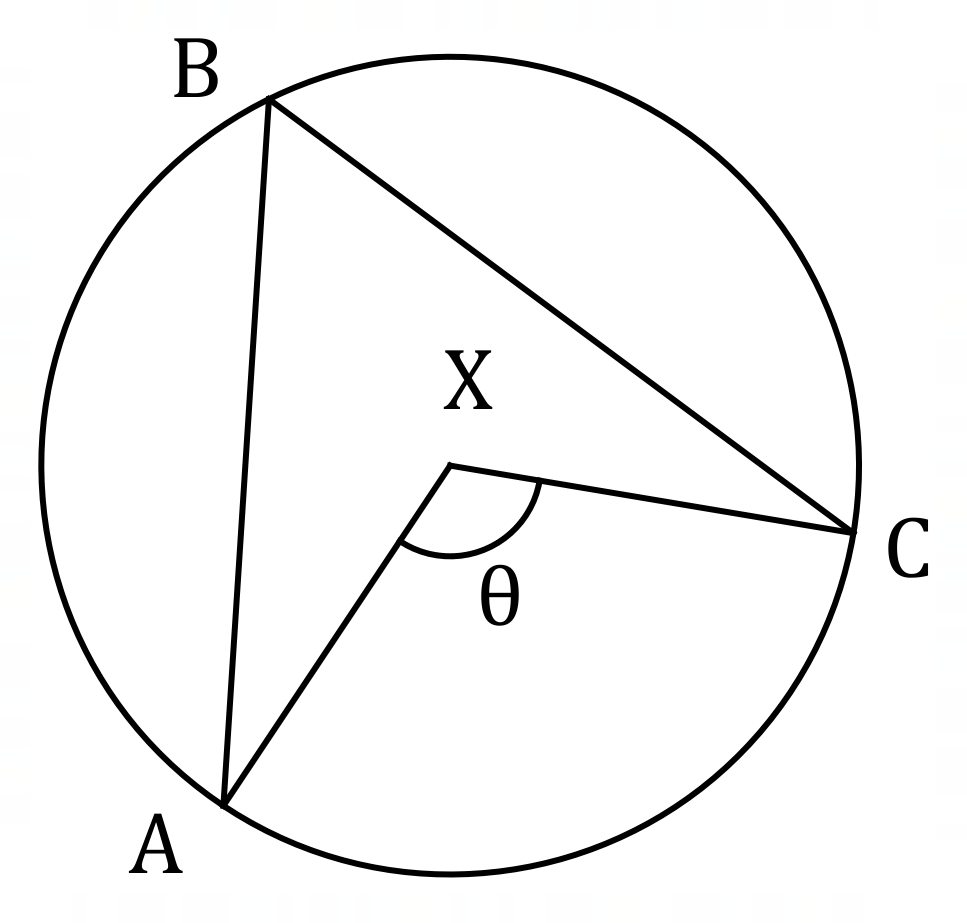
Question 16 (7 marks)

(a) The diagram shows points and on  
the circumference of a circle. Tangents to  
the circle from and meet at point .  
  
Given that , determine  
the size of and the size of .

|  |
| --- |
| **Solution** |
| Isosceles triangle:  Alternate segment:  Cyclic quadrilateral: |
| **Specific behaviours** |
| ✓ shows use of at least two relevant circle theorems  ü first angle  ü second angle |

(3 marks)

(b) In the circle shown below , and .  
Prove by contradiction that is not the centre of the circle. (4 marks)



|  |
| --- |
| **Solution** |
| Assume that is the centre of the circle.  Then (isosceles triangle)  and (isosceles triangle).  Hence and (angles at centre - circumference).  But this contradicts the initial information that and so the initial assumption must be wrong. Hence is not the centre of the circle. |
| **Specific behaviours** |
| ✓ clearly states assumption that is centre of circle  ü uses isosceles triangles to calculate  ü uses angle at centre - circumference theorem  ü notes contradiction and draws conclusion |

Question 17 (8 marks)

Each letter in the word ACRIMONIOUS is printed individually on a card. When cards are arranged next to each other in a line, determine the number of different permutations

(a) of all the cards. (2 marks)

|  |
| --- |
| **Solution** |
| Note: There are two I's and two O's. |
| **Specific behaviours** |
| ✓ expression that allows for repeated letters  ü calculates number |

(b) of all the cards where all the consonants are adjacent. (2 marks)

|  |
| --- |
| **Solution** |
| Note: There are five consonants that form a group to be arranged with the remaining six letters. |
| **Specific behaviours** |
| ✓ explains or clearly indicates grouping of consonants  ü calculates number |

(c) using any of the cards. (4 marks)

|  |
| --- |
| **Solution** |
| Consider cases by selecting and then arranging:  1. All letters different  2. One pair (II or OO) and two different:  3. Two pairs (II and OO):  Number of permutations: |
| **Specific behaviours** |
| ✓ identifies mutually exclusive cases  ü counts one case correctly  ü counts second case correctly  ü counts all cases correctly and calculates total |

Question 18 (8 marks)

Small bodies and are moving with constant velocities m/s and m/s respectively.

has initial position vector m and has initial position vector m.

(a) Determine the distance between the bodies after two seconds. (3 marks)

|  |
| --- |
| **Solution** |
| Positions after two seconds: |
| **Specific behaviours** |
| ✓ positions  ü vector  ü distance |

(b) Show that the distance between the bodies after seconds is given by .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü vector at time  ü simplifies vector  ü expression for magnitude and simplifies |

(c) Prove that the bodies do not meet. (2 marks)

|  |
| --- |
| **Solution** |
| Require :  Since the discriminant is negative, the distance can never be zero and hence the bodies never meet. |
| **Specific behaviours** |
| ✓ states condition for bodies to meet  ü justifies that condition never met |

Question 19 (7 marks)

<EFOFEX>
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FXData:
</EFOFEX>In the diagram shown, secants and   
intersect at , a point outside the circle  
with centre .

(a) Determine the size of and when and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ first angle  ü second angle |

(b) Prove that when secants and intersect at , a point outside the circle with centre , then . (4 marks)

|  |
| --- |
| **Solution** |
| Exterior angle of triangle:  Inscribed angles:  Inscribed angles:  Substituting:  Factoring: |
| **Specific behaviours** |
| ✓ relation using exterior angles  ü uses inscribed angles twice  ü substitutes and factors  ü notes reasoning throughout |

(c) Determine the size of when and . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct angle |

Question 20 (7 marks)

(a) A manufacturer makes the same plastic toy figure in different colours and sells them in packs of three. The toys inside each pack are randomly chosen from the production line in such a way that all are of a different colour.  
  
Determine the least number of packs that a retailer should buy from the manufacturer to be certain of obtaining at least four packs containing the same colour combination of toys.

(3 marks)

|  |
| --- |
| **Solution** |
| There are different packs.  Using the pigeonhole principle with the number of different packs as pigeonholes () and the number bought by the retailer as pigeons ():  The retailer must buy at least packs. |
| **Specific behaviours** |
| ✓ calculates different number of packs  ü applies pigeonhole principle  ü correct least number |

(b) A set of cards is numbered with all the integers from to inclusive. The cards are shuffled, placed face down and then the cards turned over one by one.  
  
Determine how many cards must be turned over to be certain that at least one of the numbers on a face up card will be three times the number on another face up card.

(4 marks)

|  |
| --- |
| **Solution** |
| Partition integers (pigeons) into pigeonholes, with any pair meeting given condition in same pigeonhole:  There are pigeonholes and so pigeons are required.  cards must be turned over to be certain. |
| **Specific behaviours** |
| ✓ treats integers as pigeons  ü identifies pigeonholes  ü indicates use of pigeonhole principle  ü correct number |

Question 21 (8 marks)

Harbour lies on a bearing of from harbour and the straight line distance between the harbours is km. Between the harbours, a steady current is moving in a south easterly direction at a speed of metres per second.

A boat with a cruising speed of metres per second is to travel from harbour to harbour in the least possible time.

(a) Sketch a diagram, roughly to scale, to show the resultant of the sum of the displacement vectors of the boat and the current. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows vector triangle, roughly to scale  ü labels and , sides, indicates angle |

(b) Determine the bearing it should steer, to the nearest degree, and the time its journey takes, to the nearest minute. (6 marks)

|  |
| --- |
| **Solution** |
| Boat should steer on bearing of and will reach after hours and minutes. |
| **Specific behaviours** |
| ✓ equation involving  ü solves for  ü equation involving  ü solves for  ü calculates and states bearing  ü states time, to nearest minute |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

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